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The Design of Particle Accelerators

Rui Alves-Pires [1), 2)] and *Rui Dilão* [1)]

1) Instituto Superior Técnico, Nonlinear Dynamics Group, Av. Rovisco Pais, 1096 Lisboa Codex, Portugal.
2) Universidade Independente, Av. Marechal Gomes da Costa, Lote 9, 1800 Lisboa, Portugal

Particle accelerators are machines developed to accumulate and accelerate beams of charged particles, such as protons, electrons and heavy ions. Originally developed for use in experiments for fundamental physics, the applications of accelerators have spread to other fields in industry, medicine, biology, and chemistry.

Applications of particle accelerators are based on two distinct principles: the ability to accelerate and to make collide a beam of particles with a target or with another beam circulating in the opposite direction; and the production of intense and well-tuned radiation, called synchrotron light.

Most new applications of particle accelerators in technology and applied science are based on the use of synchrotron radiation. This kind of radiation is produced when light charged particles, such as electrons and positrons, are accelerated to very high speeds, close to the speed of light, and then submitted to successive intense magnetic fields of alternating polarities. Under the action of these fields, the particles wiggle and lose energy in the form of radiation. The radiation produced in this way has high intensity, a well-defined frequency range and is finely collimated. Frequencies available range from 10^{16} to 10^{19} Hz, including X-rays, ultraviolet, visible and infrared light.

An accelerator has three basic functions: to confine, to accelerate, and to focus and keep focused a beam (or beams) of particles. Each function is implemented using a different physical principle and different technological devices. Dipole magnets, arranged along the pathway of the particles and producing uniform magnetic fields perpendicular to the trajectory, provide the curvature force necessary for the confinement of the particles in the vacuum chamber of the accelerator. Accelerating radio-frequency structures or cavities, producing varying electric fields with the direction of the trajectory, supply the accelerating energy to the particles and weakly stabilize the beam. Finally, quadrupole magnets, with a magnetic field strength varying linearly with the distance to the axis of the trajectory, deliver to the beam a first-order focusing effect.

The disposition of accelerating structures, dipoles and quadrupoles along a closed trajectory, define the essential of the optical properties of the beam in the accelerator [1]. However, despite the many well-known techniques to optimize the design of a circular accelerator, or synchrotron, there is not a single principle that can be applied in every case, and the optimization of each beam property requires a different procedure.

In general, the design, construction and optimization of a particle accelerator facility are very lengthy and complex processes done by a large team, ranging from a few to hundreds of people. In order to automate first steps of the design and optimization, we have written a package [1,2] — `Design.m` — in the programming language *Mathematica* [3] to calculate the physical parameters and the magnet layout of a circular particle accelerator.

With the features of *Mathematica*, we have the possibility of visualizing and symbolically manipulating the functions describing the optical properties of an accelerator, the package being an important tool for subsequent analysis at later stages of the design.

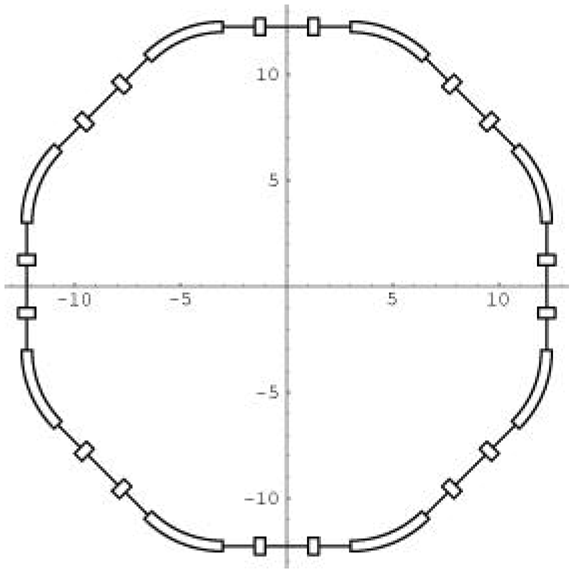
The package `Design.m` provides simple commands that allow the creation of basic accelerator layouts and its modification, for example, for adding quadrupoles at specified locations. The input parameters are the momentum of the particles, the number, length, and strength of the dipole and quadrupole magnets and the length of the straight sections. As output, the program returns basic accelerator parameters, such as mean radius, revolution frequencies, energies and energy radiated. For example, a typical output for the command

```
CreateBasicLayout[ momentum, dipole_field, nr_of_dipoles, length ]
```

where `momentum = 3000` (MeV), `dipole_field = 2` (tesla), `nr_of_dipoles = 8`, `length = 6` (meter), would be:

```
Number of Straight Sections and Dipoles:  8

Bending Angle by Dipole:    0.785398 rad
Length of Dipole:           3.92971 m
Straight Sections Length:   6. m
Maximum Dipole Field:       2. T
Cyclotron Radius:           5.00346 m

Length:       79.4377 m
Mean Radius:  12.6429 m

Maximum Energy:
Electrons:  3000. MeV   (Kinetic:  2999.49 MeV)
Protons:    3143.3 MeV  (Kinetic:  2205.03 MeV)

Revolution Frequency  (RF Frequency/Harmonic Number):
Electrons:  3.77393 MHz
Protons:    3.60188 MHz

Slip Factor:
Electrons:  0.395753;  γ = 5870.85
Protons:    0.306651;  γ = 3.3501

Energy Radiated per Turn:
Electrons:  1.43265 MeV
Protons:    1.51902 10^-13  MeV
```

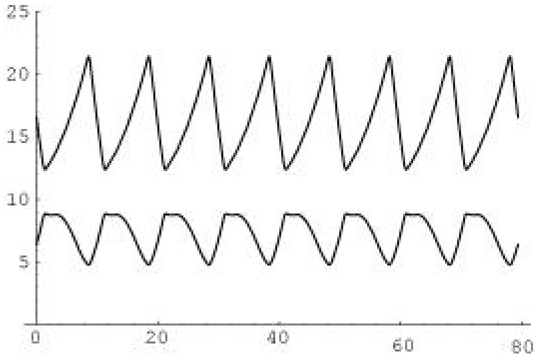
It is then possible to add quadrupoles to stabilize the transverse motion of the particles. For example, we can add pairs of quadrupoles with alternating gradients $k = \pm 0.3\ m^{-2}$ to every straight section. The quadrupoles are 0.5-meter long and at 1.5-meter from the nearest dipoles. Then we can visualize this new accelerator layout:



The stability of the lattice can also be readily obtained. For the above lattice, a single command call returns the following output, giving the horizontal and vertical one-turn, or Poincaré, maps [1] of the accelerator, as well as its tune and chromaticity:

```
MapH = {1.19865, -5.74699}
       {0.240325, -0.317979}

MapV = {-1.51023, -15.773}
       {0.265558, 2.11136}

Horizontally Stable:
       Trace[MapH] = 0.880666
       Tune        = 1.82257
       Natural Chromaticity = -1.94664
       Total Chromaticity   = -2.17915

Vertically Stable:
       Trace[MapV] = 0.601136
       Tune        = 0.798588
       Natural Chromaticity = -0.825794
       Total Chromaticity   = -0.581268
```

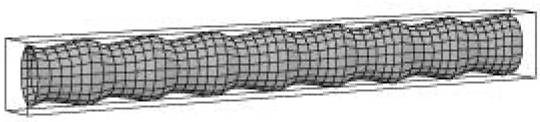
When the orbits of the accelerated particles are stable the routine also computes the optical functions describing the horizontal and vertical dimensions of the beam, and those functions can be plotted as any *Mathematica* function. For the above lattice, those functions, called beta functions, are:



With *Mathematica* is also possible to manipulate the beta functions, for example to compute the beam shape. The transverse section of the beam is an ellipsis with axis dimensions

$$\sqrt{e_H b_H} \qquad \text{and} \qquad \sqrt{e_V b_V}$$

where the β's are the horizontal and vertical beta functions and the ε's are constants called emittances. Using a correct parameterization it is then possible to plot the actual beam envelope along the longitudinal direction of the machine:



Another advantage of this package is the calculation of the exact beam chromaticity, which is an important tool to stabilize off-momentum particles.

The package `Design.m` can be obtained directly from the authors and is documented in references [1] and [2].

REFERENCES:

[1] Rui Dilão and Rui Alves-Pires, *Nonlinear Dynamics in Particle Accelerators*, World Scientific, 1996.
[2] Rui Alves-Pires and Rui Dilão, *The Design of Synchrotron Accelerators*, Mathematica in Research and Education, in press.
[3] Stephen Wolfram, *The Mathematica Book*, 3rd ed., Wolfram Media / Cambridge University Press, 1996.